1) \( f(x) = 6x^2 - x^4 \)

- **x-intercept**: \((0,0)\)
- **y-intercept**: \((0,0)\)
- **vertical asymptote**: none
- **horizontal asymptote**: none
- **relative maximum**: \((-\frac{\sqrt{5}}{15}, 9)\)
- **relative minimum**: \((0, 0)\)
- **critical points**: \((-\frac{\sqrt{5}}{15}, -\frac{2}{3})\), \((\frac{\sqrt{5}}{15}, -\frac{2}{3})\)
- **inflection points**: \((0, 0)\), \((\frac{1}{5}, 1)\), \((-\frac{1}{5}, 1)\)
- **concealed up**: \((-\infty, 0)\)
- **concealed down**: \((0, \infty)\)

**Required work**:

\[
F(x) = x^2(6-x^2)
\]

\[
F'(x) = 12x - 4x^3 + 0 = 4x(3-x^2),
\]

\[
F''(x) = 12 - 12x^2 = 0 \Rightarrow x = 0, \pm \sqrt{3}
\]

\[
F''(x) = \frac{4x}{(x^2 - 1)^2}
\]

\[
F''(x) = \frac{-4 - 12x^2}{(x^2 - 1)^3}
\]

**Note all relevant properties of \( f \) and sketch the graph (Label the maximum, minimum and inflection points)**

2) \( f(x) = \frac{-2}{x^2 - 1} \)

- **x-intercept**: none
- **y-intercept**: \((0, 2)\), \(x = -1\)
- **relative maximum**: none
- **relative minimum**: \((0, 1)\), \((-\infty, -1)\)
- **critical points**: \((-\infty, -1)\), \((1, 1)\)
- **concave up**: \((-\infty, -1)\)
- **concave down**: \((1, \infty)\)

\[
F'(x) = 0 \text{ when } x = 0
\]

\[
F'(x) = \text{undefined when } x = \pm 1
\]

\[
F''(x) = 0 \text{ when } x = 0
\]

\[
F''(x) = -4 - 12x^2 = 0
\]

\[
F''(x) = \text{undefined when } x^2 - 1 = 0 \Rightarrow x = \pm 1
\]

\[
F''(x) = \frac{4x}{(x^2 - 1)^2}
\]

\[
F''(x) = \frac{-4 - 12x^2}{(x^2 - 1)^3}
\]

**Caution points**:

- Cannot be inflection points because there are vertical asymptotes.
3) Find each indicated asymptote

a) \( f(x) = \frac{8x^3 - 2x - 5}{7x - 3} \)

\[ \text{vert. asym. } x = \frac{3}{7} \]

b) \( f(x) = \frac{5x - 7}{\sqrt{9x^2 + 8x - 2}} \)

\[ \text{horiz. asym. } y = \frac{5}{9} \]

c) \( f(x) = \frac{5x^2 - 7x + 1}{x - 3} \)

\[ \frac{31}{5} - \frac{7}{8} = \frac{24}{40} \]

oblique asym. \( y = 5x + \frac{24}{5} \)

4) Graph from \([-3, 4]\)

\( f(-3) = 1 \quad f(-2) = 3 \quad f(0) = 0 \quad f(1) = 2 \quad f(4) = 3 \)

\( f'(x) \)

\[ + - 0 + \]

\[ - \quad 0 \quad + \quad 0 - \]

\( f''(x) \)

\[ -2 \quad \frac{1}{2} \quad 1 \quad 1 \]

5) A rectangle is bounded by the x-axis and the equation \( y = \sqrt{200 - x^2} \).

a) What length and width should the region be so that its area is a maximum? 

b) What is the area? 2000

\[ y = \sqrt{(200)^2 - x^2} \]

radius = 200 \( \approx 141 \)

\[ A = \int_{-10}^{10} \left( 200 - \sqrt{200 - x^2} \right) dx \]

\[ A = 2x \sqrt{200 - x^2} + \frac{1}{2} (200 - x^2) \]

\[ x = 10 \quad \text{area} = 1000 \]

6) You have 1200 ft. of fencing and wish to fence off three adjacent rectangular fields as shown below.

a) What length and width should the region be so that its area is a maximum? \( L = 300 \text{ ft}, \quad W = 150 \text{ ft} \)

b) What is the area? \( 45,000 \text{ ft}^2 \)

\[ \text{perimeter} 6x + 4y = 1200 \]

\[ A = 3xy \]

\[ y = 1200 - 6x \]

\[ A = 3x(300 - \frac{3}{2}x) \]

\[ A' = 900 - 9x \]

\[ \frac{900 - 9x}{0} = 0 \]

\[ x = \frac{900}{3} = 100 \]

\[ y = 1200 - 6 \times 100 = 300 \]

\[ \text{length} = 3x = 3(100) = 300 \]

\[ \text{width} = 300 - \frac{3}{2}(100) = 300 - 150 = 150 \]

\[ \text{area} = 3(100)(150) = 45,000 \]

7) I need to fence off one field along a straight river. I need the area to be 1352 ft^2.

What length and width should the region be so that its perimeter is a minimum? \( x = 52 \text{ ft}, \quad y = 26 \text{ ft} \)

What is the perimeter? \( 104 \text{ ft} \)

\[ y \]

\[ A = 1352 \]

\[ \frac{\sqrt{1352}}{x} \]

\[ x = 1352 \]

\[ \rho = x + 2y \]

\[ \rho' = \frac{-1352}{y^2} + 2 = 0 \]

\[ y^2 = 676 \]

\[ y = 26 \]

\[ x = 1352 \]