Find the extreme values of \( f \) on the given interval. Determine at which numbers in the interval they occur.

1) \( f(x) = 3x^3 - 9x + 4 \); \([-2, 3]\)

- **Abs. max. value**: 58
- **Abs. min. value**: -2

**Abs. max. occurs at**
\[ x = 3 \]

**Abs. min. occurs at**
\[ x = -2 \text{ and } x = 1 \]

3) Find the relative max. and min. and the intervals on which the given function is increasing and those on which it is decreasing. \( f(x) = x(x - 10)^3 \)

- **rel. max.**: None
- **rel. min.**: -1054.688

**dec.**
\[ (-\infty, 10) \cup (10, \infty) \]

**inc.**
\[ (-\infty, 2.5) \]

4) Find any inflection point and the intervals on which the function is concave upward and those on which it is concave downward. \( g(x) = x^4 - 4x^3 + 2x + 1 \)

- **inf. pt.**
  \( (0, 1) \in (2, -1) \)
- **conc. up**
  \( (-\infty, 0) \cup (2, \infty) \)
- **conc. down**
  \( (0, 2) \)

5) From \([0, 5]\) tell me about the function. (Use graph to the right)

- **List the \( x \)-coordinates for each**:
  - Inflection points: 2
  - Relative maximum: 1, 3
  - Relative minimum: 2
  - Hard points: 1, 2

- **Find each**:
  - Abs. max. value: 3
  - Abs. min. value: -1
  - Abs. max. value occurs at \( x = 1 \)
  - Abs. min. value occurs at \( x = 5 \)

- **On which interval(s) is the graph**: increasing/concave up \( (0, 1) \)
  increasing/concave down \( (2, 3) \)
  decreasing/concave up \( (1, 2) \)
  decreasing/concave down \( (3, 5) \)

6) If \( f(x) = x^3 - 61 \), and \( x_1 = 4 \). Use Newton's Method to find the third approximation \( x_3 \).
7) \( f''(x) = (x - 2)^2 (2x + 7) \) Find where inflection point(s) occur(s) and concavity.

\[
\begin{array}{ccc}
\text{inf. pt.} & \text{conc. up} & \text{conc. down} \\
\frac{-7}{2} & (-\frac{7}{2}, 2) & (-\infty, -\frac{7}{2}) \\
& (2, \infty) & \\
\end{array}
\]

8) \( f'(x) = x - \frac{5}{x} \) Find where the rel. extreme values occur and when the graph increases and decreases.

\[
\begin{array}{cccc}
\text{rel. max.} & \text{rel. min.} & \text{inc.} & \text{dec.} \\
x = 0 & x = \pm \sqrt{5} & (-\sqrt{5}, 0) & (-\infty, -\sqrt{5}) \\
& (\sqrt{5}, \infty) & (0, \sqrt{5}) & \\
\end{array}
\]

9) The average cost of our product is given by \( \bar{C} = 10x + \frac{400,000}{x} \).

a) How many of our product should we make to minimize the average cost? \( 200 \) units

b) What is the average cost per unit? \( \$4000 \) per unit.

10) \( f'(x) < 0 \) and \( f''(x) > 0 \) \( \text{decreasing, concave up} \)

11) \( f'(x) > 0 \) and \( f''(x) < 0 \) \( \text{increasing, concave down} \)

12) \( f'(x) < 0 \) and \( f''(x) < 0 \) \( \text{decreasing, concave down} \)

13) \( f'(x) > 0 \) and \( f''(x) > 0 \) \( \text{increasing, concave up} \)

14) Draw each graph

15) Given \( f'(-2) = 0 \) and \( f'(5) = 0 \) and \( f''(x) = \frac{9x + 11}{(x+3)^3} \). Use the 2nd derivative test to determine if the critical points are relative maximums, relative minimums, or neither.

\( -2 \) is \( \text{relative maximum} \) \( 5 \) is \( \text{relative minimum} \)

16) Given \( f(x) = \frac{10}{x} \), find all numbers c in the interval \((1,5)\) where the Mean Value Theorem applies.

\[ x = \sqrt{5} \text{ or } x = -\sqrt{5} \]