Notes Ch.3 (Part 2)

Curve Sketching and Applications

Learning Objectives:
1. Determine characteristics of a function using 1\textsuperscript{st} and 2\textsuperscript{nd} derivative test
2. Graph a function by plotting any intercepts, relative extrema, holes, asymptotes. Use concavity and intervals of increasing and decreasing to correctly plot the curvature.
3. Solve real-world optimization problems.

Chapter 3 (part 2) Coursework & Agenda:

<table>
<thead>
<tr>
<th>Date</th>
<th>Class Activity</th>
<th>Homework</th>
</tr>
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<tbody>
<tr>
<td>Friday, Nov. 14</td>
<td>Test on CH. 3 Part 1</td>
<td>“Curve Sketching” Video Do Problem 6, 8, 24 on p. 215</td>
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<tr>
<td>Tuesday, Nov. 18</td>
<td>(3-7) p. 223 #21, 22, 25, 26</td>
<td>p. 224-227, #29, 31 cd, 33, 39, 40, 60</td>
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<tr>
<td>Wednesday, Nov. 19</td>
<td>Ch. 3 Review WS #2 (Curve Sketching &amp; Optimization)</td>
<td>Complete Worksheet #2</td>
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<td>Thursday, Nov. 20</td>
<td>Review</td>
<td>Study for Ch.3 Test part 2</td>
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<td>Friday, Nov. 21</td>
<td>Test Ch. 3 Part 2</td>
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3.6 Curve Sketching Review

1) X-intercepts: Where the curve crosses the x-axis
   Find by solving \( f(x) = 0 \) (let \( y = 0 \))
   **Form:** \((x,0)\)

2) Y-intercepts: Where the curve crosses the y-axis
   Find by evaluating \( f(0) \) (let \( x = 0 \))
   **Form:** \((0,y)\)

3) Holes:
   Occurs when numerator and denominator have a common zero:
   To get the y-value, simplify \( f(x) \) to remove the hole, then substitute that x-value into the reduced function.
   **Form:** \((x,f(x))\)

4) Vertical Asymptotes
   Find these AFTER you have identified all holes.
   Solve for remaining zeros of the denominator
   **Form:** \(x = \text{number}\)
   The function curve will NEVER cross the vertical asymptote!

5) Horizontal Asymptotes
   The curve will approach this asymptotes as \( x \) approaches \( \pm \infty \)
   Sometimes the curve DOES cross the horizontal asymptote!
   **Form for HA:**
   - If degree of numerator < degree of denominator then \( \text{HA at } y = 0 \)
   - If degree of numerator = degree of denominator then \( \text{HA at } y = \text{ratio of lead coefficients} \)

6) Increasing / Decreasing / Relative Extrema
   Find the critical values \((x \text{ such that } f'(x) = 0 \text{ or } f'(x) \text{ undefined})\).
   - If \( f'(x) > 0 \) then \( f(x) \text{ is increasing} \)
   - If \( f'(x) < 0 \) then \( f(x) \text{ is decreasing} \)

7) Concavity
   Find the critical values \((x \text{ such that } f''(x) = 0 \text{ or } f''(x) \text{ undefined})\).
   - If \( f''(x) > 0 \) then \( f(x) \text{ is concave up} \)
   - If \( f''(x) < 0 \) then \( f(x) \text{ is concave down} \)

Inflection points are coordinates where concavity changes sign.
Sketch the graph (Label the maximum, minimum and inflection points)

**EX#1:**  
\[ y = x^3 - 3x^2 \]  \[ y' = 3x^2 - 6x \]  \[ y'' = 6x - 6 \]

<table>
<thead>
<tr>
<th>x-int</th>
<th>y-int</th>
<th>v.asym.</th>
<th>h.asym.</th>
<th>rel.max.</th>
<th>rel.min.</th>
<th>inc.</th>
<th>dec.</th>
<th>inf.pts.</th>
<th>conc.up</th>
<th>conc.down</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>NONE</td>
<td>NONE</td>
<td>(0, 0)</td>
<td>(2, -4)</td>
<td>(-\infty, 0)</td>
<td>(0, 2)</td>
<td>(1, -2)</td>
<td>(1, \infty)</td>
<td>(-\infty, 1)</td>
</tr>
<tr>
<td>(3, 0)</td>
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\[ x^3 - 3x^2 = 0 \]  \[ 3x^2 - 6x = 0 \]  \[ 6x - 6 = 0 \]
\[ x^2 (x - 3) = 0 \]  \[ 3x(x - 2) = 0 \]  \[ 6(x - 1) = 0 \]
\[ x = 0, 3 \]  \[ x = 0, 2 \]  \[ x = 1 \]

\[ f'(x) \quad \begin{cases} + & 0 \\ - & 0 \\ + & 2 \end{cases} \]
\[ f''(x) \quad \begin{cases} - & 0 \\ + & 1 \end{cases} \]
Note all relevant properties of the function and sketch the graph. Label the Maximum (M), the minimum (m) and the Inflection points (I).

**EX#2:**

\[ y = 3x(x - 2)^3 \]

\[ y' = \]

\[ y'' = \]
EX#3:

\[ y = \frac{12x}{(x+1)^2} \]

\[ y' = \]

\[ y'' = \]
3-7 Optimization Problems

STEPS
1) Draw and label a picture
2) Write a primary equation for the quantity you are trying to maximize or minimize
3) Reduce the primary equation to a single independent variable. You may need to use an auxiliary equation that relates the independent variables.
4) Determine the feasible domain for the problem
5) Use the 1st derivative to determine maximum or minimum values
6) Find remaining information.

Ex#1:
An open box of maximum volume is to be made from a square piece of material, which is 12 inches on a side. This will be done by cutting equal squares from the corners and turning up the sides. How much should you cut off from the corners? What is the maximum volume of the box?

Ex.#2
A farmer plans to fence a rectangular pasture adjacent to a river. The farmer has 48 feet of fence in which to enclose the pasture. What dimensions should be used so that the enclosed area will be a maximum? What is the maximum area?
EX. #3
A crate, open at the top, has vertical sides, a square bottom and a volume of 256 ft$^3$. What dimensions give us a minimum surface area? What is the surface area?

EX. #4:
A rectangle is bounded by the x-axis, and the semi circle $y = \sqrt{16-x^2}$. What length and width should the rectangle have so that its area is a maximum?