1) Find \( \frac{dx}{dt} \) given \( x = 5 \) and \( \frac{dy}{dt} = 7 \) for the equation \( 3x^2 - 5y^3 = 35 \).

2) Two trains leave the station at the same time with one train traveling north at 16 mph and the other train traveling east at 30 mph. How fast is the distance between the two trains changing after 3 hours?

\[
\begin{align*}
\frac{dx}{dt} &= 16 \\
\frac{dy}{dt} &= 30 \\
\end{align*}
\]

\[
\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{16^2 + 30^2} = 34 \text{ mph}
\]

3) The radius of a circle is increasing at the rate of 4 feet per minute.

a) Find the rate at which the area \( A = \pi r^2 \) is increasing when the radius is 12 feet. \( \frac{dA}{dt} = 96\pi \text{ ft}^2/\text{min.} \)

b) Find the rate at which the circumference \( C = 2\pi r \) is increasing at the same time. \( \frac{dC}{dt} = 8\pi \text{ ft/ min.} \)

a) \[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi (12)(4) = 96\pi \text{ ft}^2/\text{min.}
\]

b) \[
\frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi (4) = 8\pi \text{ ft/ min.}
\]

4) A spherical balloon \( V = \frac{4}{3} \pi r^3 \) is inflated at the rate of 11 cubic feet per minute.

a) How fast is the radius of the balloon changing at the instant the radius is 5 feet? \( \frac{dr}{dt} = \frac{11}{100\pi} \text{ ft/ min.} \)

b) How fast is the surface area \( A = 4\pi r^2 \) of the balloon changing at the same time? \( \frac{dA}{dt} = \frac{22}{5} \text{ ft}^2/\text{min.} \)

a) \[
\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi (5)^2 \frac{dr}{dt} = 11 \text{ ft/min.}
\]

b) \[
\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi (5) \frac{11}{100\pi} = \frac{22}{5} \text{ ft}^2/\text{min.}
\]

5) The height of a cylinder with a radius of 4 ft. is increasing at a rate of 2 feet per minute.

Find the rate of change of the volume of the cylinder when the height is 6 feet. \( V = \pi r^2 h \)

\[
\begin{align*}
V &= \pi r^2 h \\
\frac{dV}{dt} &= 16\pi \frac{dh}{dt} \\
\frac{dV}{dt} &= 16\pi (2) \\
\frac{dV}{dt} &= 32\pi \text{ ft}^3/\text{min.}
\end{align*}
\]

Name: _____________________________  Per. ____
6) A conical tank is 20 feet across the top and 15 feet deep. If water is flowing into the tank at the rate of 9 cubic feet per minute, \( V = \frac{1}{3} \pi r^2 h \)

a) find the rate of change of the depth of the water the instant that it is 2 feet deep. \( \frac{dh}{dt} = \frac{81}{16\pi} \text{ ft/min.} \)

b) find the rate of change of the surface of the water at the same time. \( \frac{dA}{dt} = 9 \text{ ft}^2/\text{min.} \)

\[
\begin{align*}
\frac{dV}{dt} &= \frac{4}{9} \pi h^2 \frac{dh}{dt} \\
\frac{dA}{dt} &= \frac{8}{9} \pi h \frac{dh}{dt} \\
\frac{dA}{dt} &= \frac{8}{9} \pi \left(2 \right) \frac{81}{16\pi} \\
\frac{dA}{dt} &= 9 \text{ ft}^2/\text{min.}
\end{align*}
\]

7) A man standing on a 100 ft. cliff watches a boat heading away from the cliff. The boat is travelling at a rate of 88 ft/s.

a) How fast is the distance \( k \) between the boat and the man changing when the boat is 70 ft. from the cliff? \( \text{50.465 ft/s.} \)

b) How fast is the angle \( \theta \) changing at this time? \( \text{0.591 radians/s} \)

\[
\begin{align*}
x^2 + 10000 &= k^2 \\
2x \frac{dx}{dt} &= 2k \frac{dk}{dt} \\
2(70)(88) &= 2 \left(\sqrt{14900}\right) \frac{dk}{dt} \\
70 \frac{d\theta}{dt} &= \frac{-100}{\sqrt{14900}} \\
\frac{d\theta}{dt} &= \frac{-100}{14900} \left(\frac{6160}{\sqrt{14900}}\right)
\end{align*}
\]

8) A plane is travelling toward an observer at 300 mph. The plane is flying 3 miles above the ground.

a) How fast is the distance \( m \) between the plane and the man changing when the plane is 5 miles from the man (\( m = 5 \))? \( \frac{dx}{dt} = -240 \text{ mph} \)

b) How fast is the angle of depression \( \theta \) changing at this time? \( \frac{d\theta}{dt} = 36 \text{ radians/hr} \)